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# REVIEWS

*Edited by* KAREN HUNGER PARSHALL

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## **Nathan Jacobson: Collected Mathematical Papers.** By Nathan Jacobson. Boston, Basel, Berlin (Birkhäuser). 1989. 3 Vols. xviii + 1606 pp.

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These three volumes contain the papers—of both a research and an expository nature—published by the American algebraist, Nathan Jacobson, during the years from 1934 to 1988. In addition to a chronology and list of Jacobson's Ph.D. students and their thesis titles, the volumes include a detailed personal history and technical commentary written by Jacobson.

Most of the papers collected here concern the following four topics: Lie algebras, Jordan algebras, Galois theory, and the structure theory of rings and algebras. Present at the Institute for Advanced Study in 1934 when Hermann Weyl delivered his famous lectures on continuous (or Lie) groups, Jacobson has maintained a life-long interest in Lie theory, and especially in the theory of Lie algebras. For example, in an important paper, entitled "Restricted Lie algebras of characteristic $p$" and published in the *Transactions of the American Mathematical Society* in 1941, Jacobson invented the notion of a *restricted* Lie algebra, namely, a Lie algebra over a field of positive characteristic $p$ which carries an addition structure defined by a $[p]$-operator: $x \rightarrow x^{[p]}$. Over the years, the study of these Lie algebras, and particularly the simple restricted Lie algebras, has produced not only a rich independent literature but also significant applications to the theory of purely inseparable field extensions, to the representation theory of algebraic groups, to the theory of nilpotent groups, and even to homotopy theory.



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In fact, much of this work has been carried out by Jacobson and his mathematical descendants.

Also in Lie theory, Jacobson made significant contributions to the classification of simple Lie algebras over general fields, to the representation and structure theory of Lie algebras, and to the establishment of connections between Lie and Jordan theory. Characteristic of some of his work, his approach often involved "rational techniques," that is, methods for studying algebras without recourse to passing to an algebraically closed field. As he relates it, Weyl suggested this technique in his 1934 lectures, and it was consistent with the work Jacobson's Ph.D. advisor, Joseph H. M. Wedderburn, had presented in his famous paper [Maclagan-Wedderburn 1907] on associative algebras. Rational techniques have continued to hold an important place in the theory of Lie algebras [Seligman 1976]. Taken as a whole, Jacobson's research comprises some of the most important contributions to Lie theory in the past half-century.

Perhaps the author is best known, however, for his work (from 1945 to 1947) on the structure theory of rings without finiteness assumptions. Earlier studies of the structure theory by Wedderburn [1907], and later by Emil Artin [1927a, b] generally assumed that the ring was a finite dimensional algebra over a field (Wedderburn) or satisfied a descending chain condition on left or right ideals (Artin). Jacobson was the first mathematician to make really important progress without these assumptions. The famous Jacobson–Chevalley density theorem, which easily implies the Wedderburn–Artin structure theorem, dates from this period. Furthermore, Jacobson's discovery, published in 1945, of the Jacobson radical for an arbitrary ring marked an advance in ring theory of epic proportions. As an application of the new structure theory, consider the following beautiful theorem of Jacobson also published in 1945: If $R$ is a ring such that, for any $a \in R$, $a^{n(a)} = a$ for some integer $n(a) > 1$, then $R$ is commutative. As with his work on Lie theory, Jacobson's research on ring theory has assumed a permanent place among the day-to-day tools of many working algebraists. Also, these results have found wide applicability in many other areas of mathematics.

Another topic which attracted Jacobson's attention in the 1940s was Jordan theory. The concept of a Jordan algebra arose in the work of the German physicist, Pascual Jordan, which attempted to find a suitable formalism for quantum mechanics. An algebra over a commutative ring $F$ is called *Jordan* if it is commutative ($ab = ba$, for all elements $a$, $b$) and if, in place of the associative law, it satisfies the weaker identity $(a^2b) = a^2(ba)$. For example, an associative algebra $A$ with associative product $a \times b$ can be made into a Jordan algebra by defining a Jordan product $ab = \frac{1}{2}(a \times b + b \times a)$. (Here we assume 2 is invertible in $F$. Otherwise, the appropriate notion is that of a *quadratic* Jordan algebra.) Despite their origins in physics, Jordan algebras have found remarkable connections with exceptional geometries, Lie algebras and groups, complex analysis, and symmetric spaces. After early progress by Jordan, John von Neumann, and Eugene Wigner, the University of Chicago mathematician, A. Adrian Albert, obtained (in the late 1940s) a structure theory of finite dimensional Jordan algebras over fields of characteristic 0 as well as a classification of simple Jordan algebras in the algebraically closed case. Jacobson's work on Jordan algebras dates from this period, and fully one-third of the papers here concern Jordan theory. They contain many important results on the representation theory of Jordan algebras, the introduction of fundamental new concepts (such as Jordan and Lie triple systems), seminal work concerning the connections of Jordan theory with Lie theory, and the development of the theory of quadratic Jordan algebras.

Interspersed among the mathematical papers themselves, the author has written a "Personal History" beginning with his birth in a Jewish ghetto in Warsaw, Poland, in 1910 and ending with the publication of this work. This autobiographical account provides a highly detailed look at the life and career of this important twentieth-century scientist. The President of the American Mathematical Society from 1971 to 1973 and the Vice President of the International Mathematical Union from 1972 to 1974, Jacobson was deeply involved in many important matters on both the national and the international levels within the mathematical community, and he shares his recollections with the reader. Furthermore, as an indefatigable traveler and lecturer, Jacobson influenced the development of mathematics in the Soviet Union, India, and China, an influence he convincingly documents. Among the many other items in the "Personal History," the reviewer found the author's account of anti-Semitism in pre-World War II American academic life particularly interesting. (When Yale University hired Jacobson in 1947, he was the first Jew ever appointed in mathematics and perhaps only the second Jew ever to hold a faculty position there up to that time.) Readers should also enjoy the author's account of various departmental and institutional issues at the University of North Carolina, Johns Hopkins University, and especially Yale University during the periods of Jacobon's tenure at these schools.

The above brief remarks certainly do not do justice to the wealth of material in the three attractive volumes of these *Collected Mathematical Papers*. For professional algebraists, they serve as an invaluable resource not only for their obvious convenience but also for Jacobson's detailed technical commentary on essentially each of his research papers. Historians may feel such commentaries run the risk of idiosyncratic bias. Nevertheless, these supplemental remarks seem highly appropriate for the purely mathematical audience, and the "Personal History" is a gold mine of otherwise easily lost information for the historian.

## REFERENCES

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